3A80.10 Lissajous Sand Pendulum

Abstract

The Lissajous sand pendulum is a small cone shaped funnel suspended by three strings. The three strings balancing the cone are tied into two strings which the pendulum can swing from. These two strings are separated so they form a Y shape when suspended. The pendulum may oscillate one of two different planes or a combination of both. Because it is allowed to oscillate in two independent directions, the pendulum will follow a path which has the shape of a Lissajous curve. The exact shape of the path will be determined by the ratio of the two frequencies, the ratio of the amplitudes and the initial phase difference. If the funnel is filled with sand, it will leave a trail behind which traces out its path. This can provide a visual way to see how Lissajous curve are formed by oscillations in two dimensions.

Picture

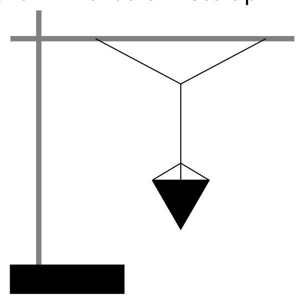
Equipment

- Lissajous sand pendulum
- Sand
- Retort stand
- Retort stand clamp
- Second retort bar
- Attachable hooks
- Shallow tray

Procedure

Attach the secondary retort bar to the stand using the clamp. The second bar should be positioned so that it is perpendicular to the first. The pendulum hangs from two strings, each with a loop at the top. The loops should be hung from the hooks which will then be attached to the second retort bar.

Figure 1: Pendulum set up



Place the tray under the pendulum. Fill the pendulum with sand and then quickly pull it diagonally so that it can oscillate in two sereate directions. Watch as the sand from the pendulum traces out the shape of a Lissajous curve.

Theory

Lissajous curves are described generally by the two parametric equations

$$x(t) = Asin(\omega_1 t + \delta),$$

$$y(t) = Bcos(\omega_2 t)$$

where A and B are the amplitudes of the waves, δ is the phase difference, and ω_1 and ω_2 are the frequency. This means that Lissajous curves can arise in any system which oscillates in two independent directions.

Table 1: Special Cases of Lissajous Curves

Curve	Amplitude	Frequency Ratio	Phase Shift
Line	Any	1	0
Ellipse	$A \neq B$	1	$\frac{\pi}{2}$
Circle	$A = B$	1	$\frac{\pi}{2}$
Parabola	Any	2	$\frac{\pi}{2}$

References

[1] Eric W. Weisstein. *Lissajous Curve*. From MathWorld–A Wolfram Web Resource. http://mathworld.wolfram.com/LissajousCurve.html, 2015.