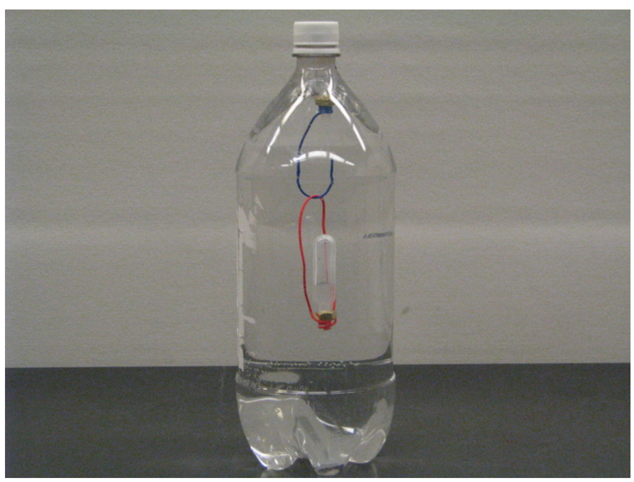
2B40.30 Cartesian Diver

Abstract

An object immersed in a fluid experiences an upward buoyant force. If the object's density is greater than the density of the fluid, the weight of the object is greater than the upward buoyant force, and the object sinks. The object will float if its density is less than that of the fluid. The average density of a pipette diver in a bottle of water increases when the bottle is squeezed because air in the pipette compresses and water flows into the pipette. As a result, the diver sinks when the bottle is squeezed.

Picture



Equipment

- diver with hook shaped wire
- diver with ring shaped wire
- 2 L plastic bottle
- water

Procedure

If the demonstration is not already assembled, place the two divers in a clear glass of water. The diver with the hook must have enough water in it so that it barely floats to the top, while the diver with the ring-shaped wire must have enough water so that it barely sinks to the bottom. Determining these quantities of water is mostly a trial-and-error process. Now place the two divers in a 2 L bottle and fill the bottle to the very top with water. Screw the cap onto the bottle and squeeze the sides. The water level in the two divers rises, and the floating diver begins to sink. If the squeeze is released, the diver with the hook rises. Hook the floating diver to the sunken diver to win.

Empty the water from the water bottle and the two divers. Wipe the parts dry and return all pieces to the demonstrations room.

Theory

A fluid is a substance that flows, such as a liquid or a gas. When an object is immersed in a fluid, there are forces exerted on the object due to the collisions of the particles in the fluid with the surface of the object. **Pressure** is defined to be,

$$p = \frac{F}{A}, \tag{1}$$

where p is the pressure of the fluid, F is the sum of all the forces applied perpendicular to the surface by the collisions, and A is the surface area of the object in contact with the fluid.

Pressure exists at all points in a fluid. However, since gravity acts downward on a fluid, it is reasonable to suspect that the pressure near the bottom of a container of fluid is **greater** than at the top. In a static fluid, as the depth of a diver increases, there is more fluid above the diver exerting weight forces on the diver, thus the pressure increases with depth. The pressure in a container of fluid is

$$p_d = p_a + \rho g d, \tag{2}$$

where p_d is the pressure at a depth d in a fluid of density ρ, g is the acceleration due to gravity, and p_a is the pressure at the surface of the fluid.

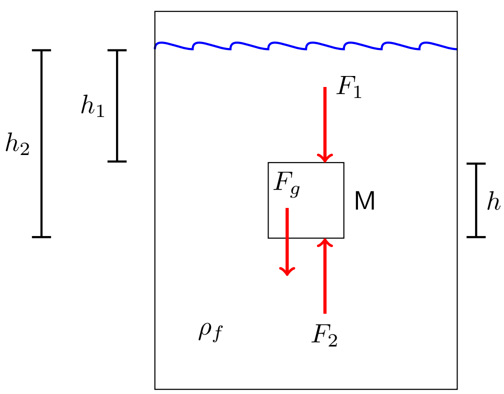


Figure 1: Diagram of a diver with mass, M, inside a container of fluid with density ρ_f. The heights h, h_1 and h_2 and the forces acting on the diver F_g, F_1 and F_2 are also labeled.

Consider an object of mass M and height h immersed in a closed container of fluid as seen in Figure 1. The density of the fluid is ρ_f and the object is in static equilibrium. Since the pressure in the fluid increases with depth, the force exerted on the bottom of the object, F_2, is greater than the force exerted on the top of the object, F_1. The net upward force between these two forces is called the **buoyant force**, that is,

$$F_B = F_2 + F_1, \tag{3}$$

where F_B is the buoyant force. According to Equation 1, the magnitude of the buoyant force can be written as

$$F_R = p_2 A - p_1 A, \tag{4}$$

where p_2 is the pressure at a depth h_2, p_1 is the pressure at a depth h_1, and A is the cross-sectional area of the object. Since the container of fluid is closed, there is no force acting downward on the top of the fluid and the term p_a in Equation 2 becomes zero. Writing the terms p_2 and p_1 in Equation 4 in terms of the variables stated in Equation 2 yields,

$$F_B = \rho_f g h_2 A - \rho_f g h_1 A = (h_2 - h_1)\, \rho_f g A = V \rho_f g, \tag{5}$$

where V is the volume of the object.

Equation 5 is **Archimedes' principle**, which states that the magnitude of the buoyant force exerted by a fluid on a partially or fully submerged object is equal to the weight of the fluid displaced by the object. In Figure 1, the buoyant force and gravity act in opposite directions and are the only forces exerted on the object in the vertical direction. If the object is in static equilibrium, then the buoyant force and gravitational force must be equal in magnitude, that is,

$$V \rho_f g = M g = V \rho_o g, \tag{6}$$

where the gravitational force is written as $V \rho_o g$ and ρ_o is the density of the object. Objects are usually composed of materials with varying densities, in which case ρ_o is the **average density** of the object. Simplifying Equation 6 yields

$$\rho_f = \rho_o, \tag{7}$$

which shows that if an object's average density is equal to the density of the fluid in which it is immersed, the object will be in static equilibrium. From similar calculations, if the object's average density is less than the density of the fluid, the buoyant force causes it to float to the surface but if the density of the object is greater than the density of the fluid the weight force causes the object to sink to the bottom.

The diver model used in the demonstration is shown in Figure 2. The density of the diver model is the average density between the plastic pipette, brass hex nut, wire, and water/air mixture in the pipette. The density of the diver can be increased by replacing the air in the pipette with water since the density of water is greater than the density of air. For reference, the density of water at STP is 0.9998676 g/cm^3 while the density of air at STP is 0.0012929 g/cm^3. If the pipette contains only air, the average density of the diver is less than the density of water and the diver floats in water. To make the diver sink, the pipette can be filled with enough water so that the average density of the diver exceeds the density of water.

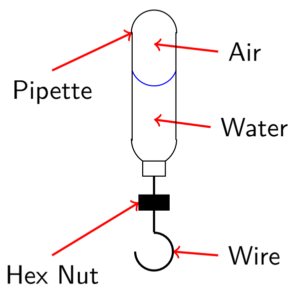


Figure 2: Diagram of the diver.

The diver with the hook is initially filled with enough water so that it barely floats in water, while the water level in the diver with the ring-shaped wire is adjusted so that it barely sinks to the bottom. It is important that the sinking diver does not have excess water to ensure that it still remains upright while at the bottom of the bottle. Both divers are placed in a sealed bottle filled to the top with water. When the bottle is squeezed, the pressure on the contents of the bottle increases. Liquids exhibit a strong resistance to compression while gases can be compressed. Following from this, the air inside the floating diver's pipette compresses and becomes denser. Since the air in the pipette now occupies a smaller volume, water can flow into the pipette, which increases the average density of the diver. If a sufficient amount of force is used to squeeze the bottle, the density of the floating diver can exceed the density of water it displaces, and the diver sinks. When the squeeze is released, the air inside the pipette expands and pushes water out of the diver, allowing it to float to the top again. The rise and drop in the water level of both divers is visible when the bottle is squeezed and released.

References

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